

Behavioural Economics and Tax Evasion: Calibrating an Agent-based Econophysics Model with Experimental Tax Compliance Data

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Abstract

We observe in the literature a persistent lack of calibrating agent-based econophysics tax evasion models. However, calibrations are indispensable to the quantitative and predictive application of such computational simulation approaches. Therefore, we analyse individual data from two tax compliance experiments with social interaction: from information on tax enforcement measures in groups with income heterogeneity, where the audit probability is known and audit results are publicly and officially announced; and from information about the mean reported income of other group members in the previous period. In our agent-based econophysics simulation, we implement recent advances in behavioural economics, for instance to describe social interactions within a population of behaviourally heterogeneous taxpayers. For this purpose, we employ experimental data showing a bimodal distribution which allows us to apply Ising's description of magnetism, a model adopted from statistical physics that can be related to binary choice models. We restrict agents in our econophysics framework to show selfish, imitating, ethical or random motives in their decisions to declare income. We find that the subjects in the experimental laboratory pursue rather mixed behaviour, including random and imitating motives.

JEL Classifications: C63; C92; H26; O17

Keywords: Tax Evasion, Tax Compliance Experiments, Agent-based Model, Behavioural Economics, Econophysics, Calibration

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INTRODUCTION

Scholars of various disciplines contribute to behavioural economics, from social scientists to physicists. Their contributions question neoclassical assumptions such as, for instance, that subjects in the experimental laboratory do not always act as

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rationally and selfishly as taxpayers in the expected utility models of Allingham and Sandmo (1972), Srinivasan (1973) and Yitzhaki (1974).⁵ Of course, in behaviourally heterogeneous populations, the tax evasion decision is embedded in a highly complex system of social interactions. Such complex systems are especially appropriate to a non-standard computational approach called agent-based modelling. This approach often succeeds in providing predictions that match real-life observations, because various kinds of interaction between autonomous agents are a common feature of agent-based models (Macal & North, 2005; Rand & Trust, 2011).

We employ statistical mechanics in an economic context to model social interactions via Ising's description of magnetism (Ising, 1925), an approach belonging to *econophysics* that combines economics and physics.⁶ In this paper, closely related to Train's (2009) and Sornette's (2014) binary discrete choice models, we apply an econophysics approach with dual aims: to analyse tax evasion behaviour, and to provide a first attempt to calibrate our agent-based econophysics model with experimental tax compliance data (Alm, Jackson, & McKee, 2009; Bazart & Bonein, 2014). Having described the theoretical model, we make use of these experimental data to test its reliability in terms of the adequacy of its theoretical and empirical findings and show its flexibility in terms of predictions, for example to identify parameter settings of interest to future experimental research. In line with previous tax compliance experiments, we confirm that social networks play an essential role in individual decisions on income declarations, and find that the majority of subjects in the experimental laboratory show a complex pattern of attitudes, mixing selfishness or ethics with imitation.

The remainder of this paper proceeds as follows. In the next section, we provide a literature overview focusing on the calibration of agent-based models of tax evasion. Next, we present our agent-based econophysics tax evasion model, including the types of behavioural agents implemented. We then briefly introduce the experimental designs of the studies used and their findings, and perform calibrations of our agent-based econophysics model with their experimental data, before making some concluding remarks.

Literature review

Agent-based tax evasion models can be categorized into economics and econophysics branches (Hokamp & Pickhardt, 2010).⁷ Zaklan, Lima, and

⁵ For a survey of tax compliance experiments in behavioural economics, see Alm (2010).

⁶ Schulz (2003) and Stauffer (2013) provide literature reviews of econophysics. Schinckus (2013) distinguishes in his overview between statistical and agent-based econophysics.

⁷ Our literature review supplements four surveys by: (i) Bloomquist (2006) on three early agent-based frameworks for tax compliance (Mittone & Patelli, 2000; Davis, Hecht, & Perkins, 2003; Bloomquist, 2004a, 2004b, 2008); (ii) Alm (2012) on recent advances in tax evasion from theory, experiments and field studies; (iii) Hokamp (2013) on agent-based tax evasion modelling; and (iv) Pickhardt and Prinz (2014) on the behavioural dynamics of tax declarations.

Westerhoff (2008) and Zaklan, Westerhoff, and Stauffer (2009) launched the econophysics branch, based on Ising's description of magnetism (Ising, 1925).⁸ One finding of these early econophysics models is that enforcement always triggers tax compliance behaviour, regardless of the prevailing social network structure. In their seminal paper, Zaklan, Westerhoff, and Stauffer (2009) find rather low rates of tax evasion for high audit probabilities. Adding the majority-vote-model and an Apollonian network, Lima (2010, 2012a, 2012b) shows the robustness of Zaklan, Westerhoff, and Stauffer (2009), thereby providing an agent-based replication study in the field of tax evasion.

Seibold and Pickhardt (2013), Hokamp and Seibold (2014b) and Pickhardt and Seibold (2014) use an agent-based econophysics approach to tax evasion based on Zaklan, Westerhoff, and Stauffer (2009) and Hokamp and Pickhardt (2010). Pickhardt and Seibold (2014) successfully replicate both underlying settings, and thus link the econophysics and economics branches of agent-based tax evasion frameworks. Seibold and Pickhardt (2013) conclude that, *ceteris paribus*, increasing the number of tax-relevant periods subject to back auditing helps to reduce tax evasion. Hokamp and Seibold (2014b) find that higher levels of public goods provision increase tax compliance. Finally, Crokidakis (2014) employs an econophysics three-state kinetic opinion exchange model to show that, above a critical threshold for the coupling of agents, tax enforcement successfully combats tax evasion. However, these econophysics studies are calibrated with neither empirical nor experimental data.⁹ Hence, we continue with a review of calibration attempts in the economics branch.

To the best of our knowledge, Bloomquist (2011a) was the first to provide a calibration of an agent-based tax evasion model. In particular, his calibration employs data from the National Research Program (NRP) of the Internal Revenue Service (IRS) for the 2001 tax year, as well as tax compliance experiments, and presents strong evidence that the attitudes to risk aversion of subjects in the experimental laboratory are similar to those of small businesses in agent-based computational simulations. Arsian and İcan (2013a, 2013b) build on Bloomquist (2011a) to conduct a tax evasion analysis for Turkey, calibrated with data from annual reports of the Turkish Revenue Administration. The authors find that von Neumann and Moore neighbourhoods are the essential social network structures to reduce tax evasion behaviour. Bloomquist (2011b) studies a synthetic county and concludes that mixed interactive auditing of heterogeneous agents is more effective than random audit strategies. Bloomquist and Koehler (2015) employ and calibrate Bloomquist (2011b), using artificial taxpayer data from Bloomquist (2012). Testing

⁸ Note that punishment in agent-based econophysics tax evasion models refers to Davis et al. (2003), in particular the notion of penalization through pre-announced time periods in which a detected tax evader has to be fully tax compliant.

⁹ Hokamp and Seibold (2014a) use aggregated experimental tax compliance data (Bazart & Pickhardt, 2011) to calculate that France seems to have a larger fraction than Germany of subjects rationally engaged in the shadow economy. Thus, they provide a calibrated agent-based econophysics model of the shadow economy.

four audit strategies, the authors show that, in terms of reducing misreported taxes, the most effective strategy is to ensure a minimum level of audits for each class of taxpayers.

Nordblom and Žamac (2012) utilize a survey of black-market service purchase in Sweden to confirm that the elderly evade substantially less tax than younger people. Miguel, Noguera, Llácer, and Tapia (2012) develop an agent-based tax evasion model for Spain to investigate behavioural mechanisms. Based on the latter model, Llácer, Miguel, Noguera, and Tapia (2013) find that considering only rational agents overestimates tax evasion, whereas social interaction allows the generation of more plausible tax compliance levels. Furthermore, Noguera, Llácer, Miguel, and Tapia (2014) calibrate Miguel et al.'s (2012) and Llácer et al.'s (2013) agent-based framework with empirical data from Spain. The authors conduct computational simulation experiments and find that social norms do not always optimize tax compliance. Garrido and Mittone (2013) calibrate their agent-based model on tax compliance data from Chile and Italy. Given income inequality, the authors find that tax authorities may optimize tax collection by auditing taxpayers who behave more frequently according to the bomb crater effect (Krauskopf & Prinz, 2011).

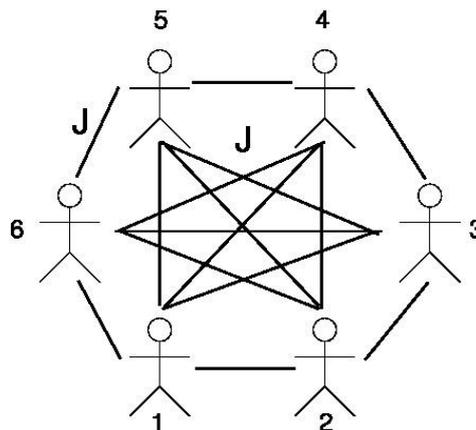
To summarize, these contributions support the relevance of modelling tax evasion decisions and social interactions in complex environments. However, we underline a persistent lack of agent-based econophysics tax evasion models regarding the purpose of calibrating computational simulations with empirical or experimental data. Such calibration might reveal both the theoretical validity and the predictive power of this tool. In the above review, we have identified six calibrated agent-based tax evasion frameworks: (i) Bloomquist (2011a) with IRS NRP and experimental data; (ii) Bloomquist (2011b) and Bloomquist and Koehler (2015) with artificial and IRS NRP data; (iii) Nordblom and Žamac (2012) with survey data from the Swedish Tax Agency; (iv) Miguel et al. (2012), Llácer et al. (2013) and Noguera et al. (2014) with Spanish empirical data; (v) Arsian and İcan (2013a, 2013b) with data from the Turkish Revenue Administration; and (vi) Garrido and Mittone (2013) with experimental data from Chile and Italy. In contrast to these calibrations based on the economics branch of tax evasion simulation models, our aim is to calibrate an agent-based econophysics model, which we present in the next section.

The agent-based econophysics approach

Within our theoretical framework, we adopt a simplified perspective on the description of tax evasion, namely that taxpayers are agents who choose between two alternatives: to declare either all or zero income to the tax authorities. The reduction of a continuous variable (i.e. the income declaration) to a binary variable may seem a drastic simplification; however, we demonstrate below that this kind of behaviour is frequently found in tax compliance experiments, and even emerges

in data from the IRS NRP for small business filers in tax year 2001 (Bloomquist, 2011a). Hence, our formal description within the Ising model, adopted from physics, corresponds with a model of discrete choice, the so-called logit binary choice model (Train, 2009; Sornette, 2014). The econophysics formulation has the advantage of providing a simpler theory structure, especially for the case of interacting agents, i.e. taxpayers. In any case, all physical quantities that appear in this model have a one-to-one correspondence in the economic language, as will be detailed below.

Figure 1. Sketch of a network considered within our econophysics approach



Note: With reference to Bazart and Bonein’s (2014) tax compliance experiment, each group consists of $N = 6$ agents which are mutually connected by an exchange coupling, J .

The Ising-model Hamiltonian

$$(1) H = -J \sum_{ij} S_i S_j - \sum_i B_i S_i$$

describes the coupling of Ising-variables (spins) $S_i = \pm 1$ between group members (labelled with $i = 1, \dots, N$). For instance, Figure 1 shows the social network for $N = 6$ agents (Bazart & Bonein, 2014). The coupling strength, J is taken as a constant between group members, and we note that each pair (ij) is only counted once.

In the present context, we interpret $S_i = +1$ ($S_i = -1$) as a compliant (non-compliant) agent. Equation (1) also contains the coupling of the spins with a local magnetic field B_i , which may be associated with agents’ moral attitudes.¹⁰ In addition, our econophysics model contains a local temperature, T_i which measures the susceptibility of agents to external perturbations (either influence of neighbours or magnetic field). We then use the heat-bath algorithm to evaluate the statistical averages of the model (Krauth, 2006). The probability of a spin at lattice site i taking values $S_i = \pm 1$ is given by

¹⁰ Note that our modelling of moral attitudes corresponds with parameter γ_i in Nordblom and Žamac’s (2012) agent-based theory.

$$(2) \quad p(S_i) = \frac{1}{1 + \exp\{-[E(-S_i) - E(S_i)] / T_i\}}$$

and $E(-S_i) - E(S_i)$ is the energy change for a spin-flip at site i . On picking a random number $0 \leq r \leq 1$, the spin takes the value $S_i = +1$ when $r < p_i$ ($S_i = +1$), and $S_i = -1$ otherwise. Obviously, one tax-relevant period then corresponds with a sweep through all members of all categories or groups.

Equation (2) has the same form as the decision probability in the logit discrete choice model, which allows for a mutual mapping of the corresponding quantities. In particular, by rewriting Equation (1) in the form

$$(3) \quad H = - \sum_i \{J \sum_j S_j + B_i\} S_i = \sum_i E(S_i)$$

it turns out that the energy $E(S_i)$ of the ‘Ising’ system corresponds with the negative observable part of the utility function for the agent at site i . This agent will choose the alternative which maximizes her utility (i.e. lowers the energy). This utility function has two contributions, a term $\sim B_i$ which reflects the endogenous (moral) attitude of the agent towards evasion, and a second term $\sim J \sum_j S_j$ which captures the influence of the agent’s social environment. The utility function is then maximized as the agent gets closer to the declaration behaviour of her neighbours in the network. Moreover, the temperature parameter T_i can, again by analogy with logit discrete choice, be interpreted as the standard deviation of the unobserved utility part corresponding with the spread in the non-measurable taste or attitude. Table 1 summarizes the parameters of the agent-based econophysics tax evasion model and compares their interpretation within the physical and economic contexts.

We then implement an enforcement scheme in our model, reflecting a case where the detection of an evading agent enforces tax compliance over the following h tax-relevant periods (or time steps). Zaklan, Lima, and Westerhoff (2008), Zaklan, Westerhoff, and Stauffer (2009), Lima (2010), Pickhardt and Seibold (2014) and Hokamp and Seibold (2014a, 2014b) invoke such a procedure, whereas Lima and Zaklan (2008) implement a randomized variant.

Table 1. Parameters of econophysics model and interpretation in physical and economic contexts

Variable	Physical Meaning	Economic Interpretation
S_i	Magnetic Moment at Position i	Decision Alternatives of Agent i : $S_i = +1$: compliant $S_i = -1$: non-compliant
B_i	Magnetic Field at Position i	Parameterized Moral Attitude of Agent i $B_i > 0$: Moral Behaviour $B_i < 0$: Amoral Behaviour
T_i	Local Temperature at Position i	Variance in Attitude of Agent i
J	Exchange of Energy between Magnetic Moments	Social Interaction Parameter between Agents
$E(S_i)$	Effective Energy for Spin at Position i	Negative Observable Part of the Utility Function for Agent i
H	Total Energy of the System	Negative Observable Part of the Aggregated Utility Function

Note: Parameters of the econophysics model described in Equations (1) and (3), and their interpretation within the logit discrete choice model applied to a binary model of tax evasion. The interaction parameter J is set to $J \equiv 1$, and therefore defines the scale for all other parameters.

Furthermore, Seibold and Pickhardt (2013) study generalizations of the auditing scheme to include time lapse effects. We set our audit probability to $p_a = 2/5$ and $p_a = 1/3$, corresponding with the values used in the tax compliance experiments used to calibrate our model (Alm, Jackson, & McKee, 2009; Bazart & Bonein, 2014).

Based on Seibold and Pickhardt (2013), Pickhardt and Seibold (2014) and Hokamp and Seibold (2014a,b), and following Hokamp and Pickhardt (2010), we assume that taxpayers can be classified into four categories of agent: (1) selfish a-type agents, which take advantage of non-compliance and are thus modelled via the parameter ratios $|B_i|/T_i \gg 1$ and $|B_i|/J \gg 1$ with $B_i < 0$; (2) imitating b-type agents, which conform to the norm of their social network, which in the model is realized through $B_i = 0$ and $J/T_i \gg 1$; (3) ethical c-type agents, which have large moral doubts and thus are practically always compliant, with behaviour parameterised by $|B_i|/T_i \gg 1$ and $|B_i|/J \gg 1$ and $B_i > 0$; and (4) random d-type agents, which act by chance, within a certain range, due to confusions caused by tax law complexity, with behaviour modelled by $B_i = 0$ and $J/T_i \ll 1$.

In the next section, we present the two tax compliance experiments (Alm, Jackson, & McKee, 2009; Bazart & Bonein, 2014) used with a view to exploring the composition of these four behavioural categories of agents.

Experimental design

In this section, we present the tax compliance experiments used to calibrate our econophysics model, the main parameters of which are reported in Table 2.

Table 2. Experimental settings used for our calibrations

Source	Alm, Jackson, & McKee (2009)	Bazart & Bonein (2014)
Treatment	Official Information (T2A)	Horizontal Inequity (H-I)
Income	Heterogenous: 100, 90, 80, 70, 60	Homogenous: 100
Tax Rate	35%	30%
Audit Probability	2/5	1/3
Fine	150%	350%
Auditing Information	Provided	Not Provided
Social Information	Not Provided	Provided
Groups	5	8
Group Size	8	6
Rounds	15	20
Total Number of Observations	600	960

Note: Auditing information means public and official announcements of audit results. Social information reflects the individual knowledge of the mean reported income of other group members.

Since social interaction is the crucial ingredient of our agent-based econophysics tax evasion simulation, it is necessary to focus on tax compliance laboratory experiments allowing social interactions. To this end, we use experimental data from Alm, Jackson, and McKee's (2009) study, which allows social information by

providing information on audits, and data from Bazart and Bonein (2014), in which social interactions are introduced through the provision of information on the average declaration of other group members.

Alm, Jackson, and McKee (2009) analysed the effect on tax compliance behaviour of dissemination of information on audit frequency and results. For this purpose, they implemented a pure declaration game in which subjects first performed a real-effort task in order to earn their income.¹¹ Next, they had to report their income for taxation at a 30 per cent rate. Because earned income was private information, any of them could under-report and decrease their tax burden. A random audit procedure was thus implemented to detect evasion that might result in the reimbursement of due tax plus payment of a penalty at 150 per cent. Subjects were placed in groups of six or eight individuals, but they did not know with whom they were playing during the 30 periods of the declaration game. To avoid cross-effects, redistribution was excluded, and to avoid end-of-game effects, participants were not informed of the exact number of periods in the declaration game. Alm, Jackson, and McKee (2009) implemented six treatments in a between-subjects design that differed depending whether and what type of information on audits was provided to the subjects. In the first three treatments, the audit probability was known to the subjects (Case A). These treatments differed in the announcement of audit results (no public announcement in T1A and T3A; public announcement in T2A) and unofficial communication (no communication in T1A and T2A; communication in T3A). The remaining three treatments were symmetric (Case B) but did not allow for an announcement of the audit probability. Unofficial communication was organized by allowing participants to send one message per round to all members of their group, mentioning whether they had been audited or caught cheating. Subjects' earnings were evaluated using all periods of the tax declaration game. The results support the positive effect of information on deterrent tools, more specifically when subjects have prior knowledge of the audit probability. For our purpose of calibrating an agent-based econophysics tax evasion model, we restricted ourselves to using the data in treatment T2A because this setting provided the maximum official information: both the audit probability and the audit results were announced. In addition, we used only the declaration choices made for each period by subjects facing an audit probability equal to $p_a = 2/5$. This reduced the sample to 40 subjects (five groups of eight subjects) for 15 periods, resulting in 600 declaration decisions.¹²

The second set of experimental data was drawn from Bazart and Bonein's (2014) study that introduced social interaction between subjects through the provision of information on the average declaration of other group members. The benchmark treatment in Bazart and Bonein (2014) was a pure declaration game, excluding redistribution through the provision of public goods financed by tax payments. In

¹¹ The earning procedure generated heterogeneity in income.

¹² The audit probability changed once at period 16.

this way, redistribution outcomes could not influence conditional reactions to others' declaration decisions. Groups of six subjects were formed and the group composition remained constant throughout the tax game. To avoid complex comparisons, all members of a group had the same income and faced the same fiscal policy parameters (i.e. tax, audit and penalty rates). At the beginning of each tax-relevant period, subjects were presented with a screen informing them of their individual income and the tax policy parameters, which were set such that they delivered the theoretical predictions of full compliance. At the beginning of each period, subjects received a constant income of 100 points each and faced: (i) a penalty rate of 350 per cent (including reimbursement of due taxes plus the fine); (ii) an audit probability of $p_a = 1/3$ (audits were random and perfect); and (iii) a benchmark tax rate of 30 per cent. At the time when they made their decisions, the subjects had to determine the amount of income they would self-report to the tax authorities. From this setting, Bazart and Bonein implemented six treatments in a between-subjects design¹³ to take into account two sources of unfairness: tax rules, and others' evasion through the provision of information on fellow citizens' average declarations. The experimental treatments were the following: (i) a benchmark treatment, in which subjects were not provided with any kind of information about the declaration of other group members; (ii) two vertical inequity treatments in which tax rates differed for fiscally identical taxpayers (being either higher or lower than the benchmark rate) but no social information was provided to subjects; (iii) a horizontal inequity treatment in which social information on the average declaration of other group members was provided;¹⁴ and (iv) two additional treatments in which vertical and horizontal inequities co-existed.

A total of 288 subjects participated in the experiment, with 24 subjects per session who repeated the declaration game over 20 periods systematically. Nevertheless, in order to calibrate our econophysics model, we needed a homogeneous set of data in which taxpayers of the four types could coexist. Consequently, we restricted the sample to the horizontal inequity treatment only, and used the declaration choices made at each period over the 20 periods of the game by the 48 subjects pertaining to this treatment, representing a total of 960 declaration decisions. Bazart and Bonein (2014) showed that some taxpayers did change their declaration decisions in the next period, to get closer to the average reported income of other group members. This behaviour was qualified as reciprocal, in that it was conditional on what the others did. Bazart and Bonein (2014) demonstrated that both horizontal positive and negative reciprocity were at stake in the experiment, meaning that, if the other group members declared more (or less) on average, the subject would increase (or decrease) his report. This is classified here under the imitating type, with the difference that a taxpayer of the imitating type will have an invariant behaviour toward copying what the others do. For this reason, the imitating type of taxpayer in our econophysics model should adjust his behaviour to that of the other

¹³ For a detailed description of the design, see Bazart and Bonein (2014).

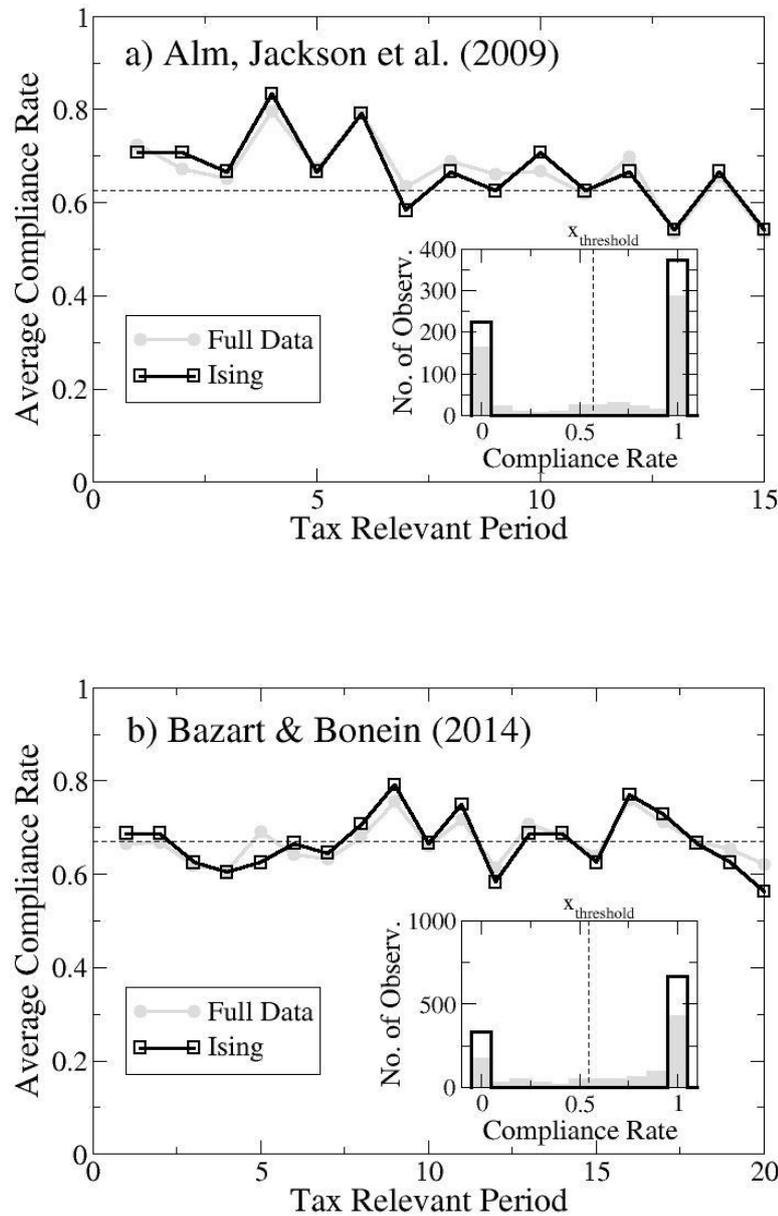
¹⁴ Horizontal inequity results from the heterogeneity of declaration decisions in the group.

group members in all 20 periods of the game. To avoid any bias linked with the history of gains in this income declaration game, the subjects' payments corresponded with the gains of five periods randomly drawn from the 20 tax-relevant periods.

In the next section, we analyse the experimental data in the settings shown in Table 2 to extract temperature and field parameters for participants in our agent-based econophysics tax evasion model.

Calibrations

Figure 2. Main panels: Comparison between Ising data (squares) and experimental data (full points) for the average compliance rate.



Note: Panel (a) refers to Alm, Jackson, and McKee's (2009) dataset, and panel (b) is for Bazart and Bonein's (2014) dataset. The horizontal dashed line corresponds with the average over time (average compliance rate of 0.62 for Alm, Jackson, and McKee, 2009, and 0.67 for Bazart & Bonein, 2014). The insets report the frequency distribution (number of observations) of the reported income over all periods compared with the bimodal Ising distribution. The vertical dashed line marks the threshold ($x_{\text{threshold}} = 0.57$ for Alm, Jackson, and McKee, 2009, and $x_{\text{threshold}} = 0.55$ for Bazart and Bonein, 2014), which is used to convert real data into Ising data.

As outlined previously, at first glance it seems a severe simplification to consider 'Ising' agents that declare either zero or full income, but data from tax compliance laboratory experiments (Alm, Jackson, & McKee, 1992; Alm & McKee, 2006; Alm, Denkins, & McKee, 2009; Bloomquist, 2011a; Alm, Bloomquist, & McKee, 2015), as well as data from the IRS NRP for small business filers, support a bimodal distribution of the compliance rate which peaks at zero and full income. The same effect is observed in Alm, Jackson, and McKee's (2009) and Bazart and Bonein's (2014) experimental data (see insets to Figure 2, 600 and 960 observations, respectively), which show major peaks in the frequency of the compliance rate at 0 and 1. In Alm, Jackson, & McKee's (2009) data (for audit probability $p_a = 2/5$), this kind of behaviour is even more pronounced.

We now adopt the following procedure to transform the experimental data x_{data} to Ising data x_{ising} . A declaration $x_{\text{data}} \leq x_{\text{threshold}}$ is taken as $x_{\text{ising}} = -1$ (zero declaration), whereas for $x_{\text{data}} > x_{\text{threshold}}$ we set $x_{\text{ising}} = 1$ (full declaration). $x_{\text{threshold}}$ is chosen, such that we obtain the same average compliance rate, averaged over periods and participants (~0.62 and ~0.67 for Alm, Jackson, & McKee's, 2009 and Bazart & Bonein's, 2014 experiments, respectively) for the experimental and Ising data. This average is marked by the horizontal dashed line in the main panels of Figure 2. As a result, we obtain $x_{\text{threshold}} = 0.57$ for Alm, Jackson, & McKee's (2009) data, and $x_{\text{threshold}} = 0.55$ for Bazart and Bonein's (2014) data. Moreover, it can be seen from the main panels of Figure 2 that the temporal evolution of both datasets is very close, which further validates our mapping procedure.

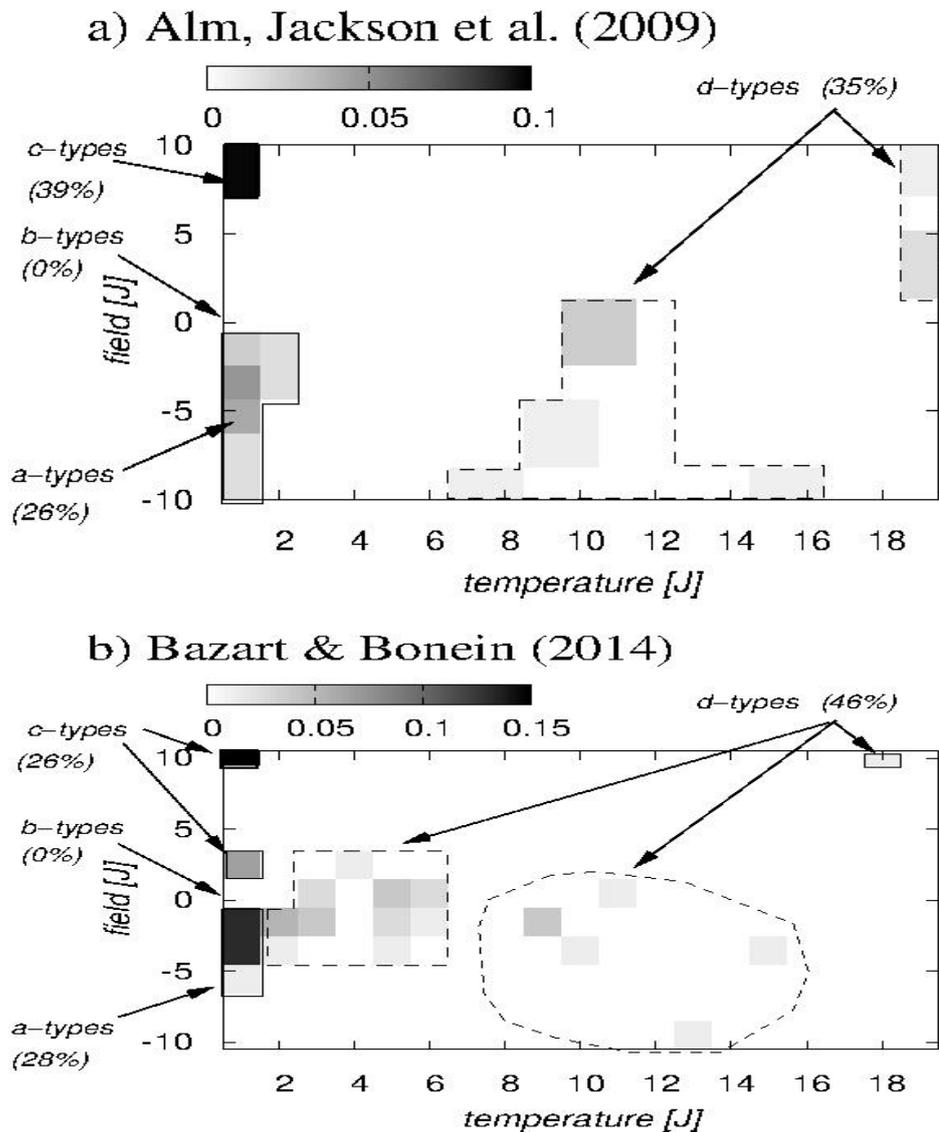
Based on the Ising dataset, we determine a local temperature T_i and magnetic field B_i parameter for each participant. For this procedure, we use Equation (2), which determines the probability $p_i = p_i(T_i, B_i)$ for the transition S_i to $-S_i$ for agent i . Since p_i depends on the state of neighbours, we first collect, for each participant in a given state S_i with a given neighbour configuration, the number of periods where this arrangement is the same. We then check whether or not the agent has changed her state in the next period, which allows for determination of the transition probability for a fixed neighbour configuration. Since we need two equations to determine the two variables T_i and B_i , we repeat the same procedure for another neighbour configuration. In practice, we take those neighbour configurations which occur most frequently within the time period of the experiment.

In this way, local temperature and magnetic field parameters are determined for each participant, and in Figure 3 we show the resulting distribution of parameter values for the data from Alm, Jackson, and McKee (2009) in Panel (a) and from

Bazart and Bonein (2014) in Panel (b). There are several notable points. First, there is no indication of a pure imitating b-type which, as noted earlier, is specified by $B_i = 0$ and $T_i \ll J$. Second, in both datasets, the percentage of a-types is comparable (26 per cent in Alm, Jackson, & McKee, 2009, and 28 per cent in Bazart & Bonein, 2014, respectively) and the majority of selfish a-types are of the same order as the interaction energy coming from nearest neighbours. Therefore, these types are not purely non-compliant but have a significant tendency to copy the behaviour of their social network. The same holds for the low-temperature d-types (~26 per cent) in Bazart and Bonein's (2014) data, which probability is also not purely random but also influenced by the state of nearest neighbours. On the other hand, we see from Figure 3b) that there is a second group of high-temperature d-types (~ 20 per cent) which, in all periods, make purely random decisions between compliance and non-compliance. The same holds for the 35 per cent of d-types in Alm, Jackson, and McKee's (2009) data.

Analysis of the agent distribution, shown in Figures 3a) and 3b), reveals interesting differences. This concerns, in particular, the percentage of c- and d-types, while the shares of (pure) b-types (0 per cent) and a-types (~26-28 per cent) are similar. In fact, the 35 per cent d-types and 39 per cent c-types in Alm, Jackson, and McKee's (2009) data appear as 46 per cent d-types and 26 per cent c-types in analysis of Bazart and Bonein's (2014) experiment, which needs explanation. In both experiments, participants were drawn from a pool of undergraduate students. Although there may have been differences in their sociocultural background (European versus US) and there is also a slight difference in audit probabilities (2/5 versus 1/3), this does not account for the difference of 10 per cent in the c- and d-type compositions. A rather more plausible explanation relates to the experimental design concerning the income of participants. While, in Alm, Jackson, and McKee's (2009) experimental design, participants earned income through their performance in a task based on 20 periods, the setting of Bazart and Bonein (2014) was such that individuals in each period received a constant income of 100 points and were paid for five randomly-selected periods. It is likely that income resulting from labour rather than as a "lump sum" was valued more highly because a high wage in one period did not guarantee the same wage in the next period. Therefore, individuals may have been more careful in managing their assets, which was in turn reflected in the increased moral attitude of the participants. On the other hand, participants who were always sure of receiving the same fixed wage in the next period may have been more susceptible to evading part of this income in order effectively to increase their assets. Such sporadic evading behaviour is characteristic of d-types, which may explain their larger percentage in Bazart and Bonein's (2014) experiment. It would be interesting to investigate this hypothesis in a future experimental study.

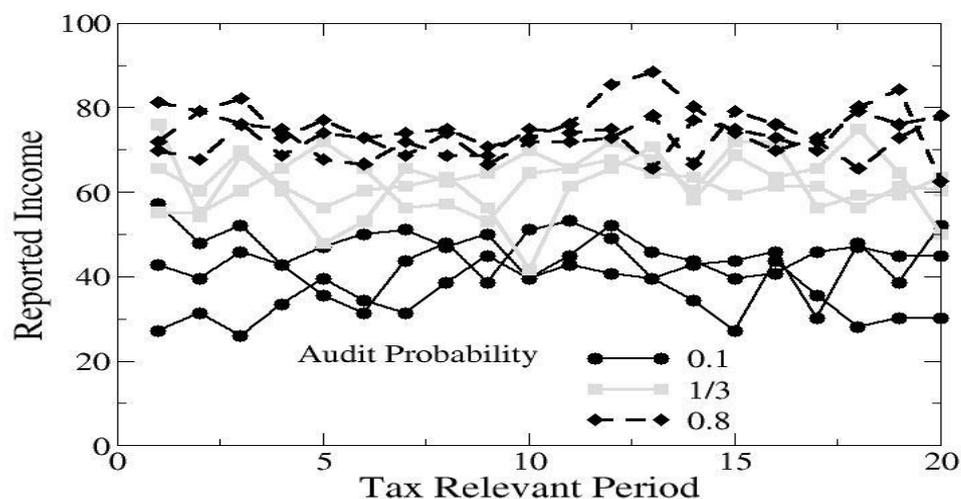
Figure 3. Parameter distribution of agent types determined from the Ising data



Note: The borderlines group the agents according to the classification of types. Panel (a) shows data from Alm, Jackson, and McKee (2009); Panel (b) shows data from Bazart and Bonein (2014).

Having characterized all participants in the tax compliance experiment by their local temperature and magnetic field parameters, we are now in a position to evaluate and predict the time-dependent reported income for different experimental settings. In Figure 4, this is exemplified for a hypothetical experiment which differs from Bazart and Bonein (2014) only in the audit probability. For each audit probability, we show three simulations which differ within the error induced by the finite group size. The interesting finding concerns the relatively small increase in reported income, from ~40 per cent to ~78 per cent, on increasing the audit probability from $p_a = 0.1$ to $p_a = 0.8$. The reason for this weak dependence on p_a can be traced back to the large fraction of d-types (~46 per cent) among the participants. Since they predominantly declare randomly, these agents are only weakly affected by an audit. Of course, this conclusion only holds when the distribution obtained in Figure 3 itself only weakly depends on audit probability.

Figure 4. Results for time-dependent reported income evaluated against the agent distribution reported in Fig. 3a and different audit probabilities



Note: For each audit probability, three simulations were realized, which differ due to finite size fluctuations.

CONCLUSIONS

In this paper, we have presented calibrations of our agent-based econophysics tax evasion model based on Pickhardt and Seibold (2014), with experimental tax compliance data taken from Alm, Jackson, and McKee (2009) and Bazart and Bonein (2014). To the best of our knowledge, this kind of calibration has never been done before in the econophysics branch of agent-based tax evasion modelling. Moreover, following the discussions in Schulz (2003), Zaklan, Westerhoff, and Stauffer (2009), Hokamp and Pickhardt (2010) and Pickhardt and Seibold (2014), we have given an economic interpretation of physical quantities in econophysics. For instance, magnetic fields reflect a moral attitude of agents, and local temperature measures the susceptibility of agents to external perturbations. According to our analysis, the pure agent types introduced in Hokamp and Pickhardt (2010) and Pickhardt and Seibold (2014) are not visible in participants in the tax compliance experiments conducted by Alm, Jackson, and McKee (2009) and Bazart and Bonein (2014). Rather, we find agent types whose behaviour is a mixture of non-compliant and imitating (a-types), compliant and imitating (c-types), and random and imitating (d-types). Only for the d-types, there also exists a pure sub-group with a large temperature parameter, so that agents act purely randomly over all periods.

Furthermore, we have been able to replicate findings frequently observed in tax compliance experiments, in particular regarding wide fluctuations in subjects' income declaration behaviour. This result is due to downsizing the population of our agent-based econophysics model (from 10^6 to fewer than 50 agents). Cline, Bloomquist, Gentile, Koehler and Marques (2013) and Bloomquist and Koehler

(2015) conduct their research in the contrary direction; that is, they build a large-scale agent-based model of tax compliance ($\sim 10^8$ agents). Our findings support their notion that scale influences aggregate taxpayer behaviour in computational social simulations. Within our agent-based econophysics approach, these differences between large- and small-scale tax evasion simulations are due to (i) enhanced statistical fluctuations relating to small group sizes, and (ii) alterations in social network structures regarding small-scale experimental designs and large-scale *real world* situations.

However, the calibrations carried out in this paper are only a first step toward establishing an agent-based econophysics approach to tax evasion dynamics. In particular, it is important to analyse whether, in our approach, the agent-type distribution is dependent on the experimental setting, for example whether it depends on audit probability. In addition, it may be that agents change their behaviour over time, so that the local temperature and magnetic field parameters acquire a temporal dependence. Further research is required to allow the forecasting of tax evasion through agent-based modelling.

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